Quantity and quality of diagrams used in math word problem solving: A comparison between New Zealand and Japanese students[1]

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Abstract

It is generally considered that diagram use aids efficacy of math word problem solving. While understanding diagrams is considered important in both New Zealand and Japanese secondary schools, there is an additional emphasis in New Zealand schools for students to appreciate their use as tools for problem solving and communication. This study examined whether there are actual differences in the amount and quality of diagrams that students in New Zealand and Japan use when given math word problems to solve. The participants were 614 secondary school students from New Zealand and Japan, aged 13 to 15 years old, who were given one- and two-object math word problems (involving length or non-length components) to solve. The findings were that while the New Zealand students evidenced greater diagram use and provided more correct answers, they produced significantly more high quality diagrams only for the easiest problems given (the one-object problems with length story context). One implication of the findings is that greater emphasis needs to be placed on, and/or more effective strategies need to be used in, developing New Zealand students' skills in the use of diagrams for solving more complex types of problems.

Introduction

Using diagrams is generally considered by teachers and researchers as one of the most effective ways of improving efficacy in solving math word problems. A meta-analysis carried out by Hembree (1992) showed that using diagrams was the most efficient of the different strategies that had been suggested as helpful for problem solving. Numerous studies (e.g., Cheng, 2004; Mayer, 2003; Novick & Hurley, 2001) have reported empirical evidence indicating the beneficial effects of diagrams or visual representations on problem solving. The usefulness of diagrams can probably be best understood in terms of Larkin and Simon's (1987) explanation that diagrammatic representations are computationally more efficient than sentential representations because they minimise labels, gather related information in one place, and promote easier recognition of the overall situation being dealt with. Thus, diagrams help reduce memory, computational, and searching loads in problem solving.

Internationally the development of skills relating to diagram use is usually included in the math curricula. In Japan, for example, the national curriculum emphasises the crucial nature of understanding diagrams and targeted mathematical concepts (Japanese Ministry of Education, 1998). Diagrams are likewise emphasised in the New Zealand (NZ) math curriculum which has, as one of its objectives, the provision of opportunities for students to "develop the ability to think abstractly and to use symbols, notations, and graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalisations" (NZ Ministry of Education, 1992, p. 10).

There are some important differences, however, between the Japanese and NZ math curricula in the way they view diagrams. One of the most important of these differences is that the NZ curriculum goes beyond simply teaching students how to understand diagrams (which is the main emphasis in the Japanese curriculum). As noted above, the NZ math curriculum also stresses the use of diagrams as a communication tool. It states that:

> Learning to communicate about and through mathematics is part of learning to become a mathematical problem solver and learning to think mathematically. Critical reflection may be developed by encouraging students to share ideas, to use their own words to explain their ideas, and to record their thinking in a variety of ways, for example, through words, symbols, diagrams and models. (NZ Ministry of Education, 1992, p. 11)

To this end, the NZ curriculum points out that teachers can create opportunities for students to develop the characteristics of good problem-solving techniques (which include both convergent and divergent approaches) "by encouraging [students] to practise and learn" various simple strategies which include "drawing a diagram" (NZ Ministry of Education, 1992, p. 11).

Notwithstanding the apparent importance of skills relating to diagram use in the school setting, various studies have described serious deficiencies in students' abilities to use diagrams when problem solving, suggesting that students generally do not appreciate the value of diagram use and/or cannot effectively use them. Observations of students' actual work in problem solving have shown, for example, that students lack spontaneity in using diagrams (e.g., Dufour-Janvier, Bednarz, & Belanger, 1987; Ichikawa, 1993, 2000; Uesaka, 2003), that they are generally poor in choosing diagrams to use, and that they fail to draw appropriate inferences when they use diagrams (e.g., Cox, 1996; Uesaka, 2003).

An important question that this raises is: to what extent might the math curriculum – and hence the ensuing learning views imparted and teaching approaches utilised – impact on student efficacy in using diagrams for problem solving? Both Japan and NZ were ranked very high in PISA (Program for International Student Assessment, a project commissioned by the Organization for Economic Co-operation and Development [OECD] which assesses the knowledge and skills of 15-year-old students from the participating industrialised countries) 2000 and PISA 2003 (OECD, 2004). However, based on the two countries' respective math curricula, one could predict that NZ students would demonstrate greater spontaneous use of diagrams when attempting to solve math word problems.

To the present authors' knowledge, no previous investigations have been carried out to confirm or refute such a prediction. Thus, the purpose of the research study reported here was to examine the spontaneous use of diagrams by students in NZ and Japan when attempting to solve math word problems. Apart from the frequency of spontaneous diagram use, the correctness of answers provided, as well as the quality of diagrams constructed by the students were examined. It was hoped that insights gained through the examination of similarities and differences in performance would contribute to the development and further refinement of effective teaching practices in both countries.

Method

Participants

Six hundred and fourteen (614) secondary school students from New Zealand and Japan, aged 13 to 15 years old (mean ages: New Zealand = 13.97 years, Japan = 13.28 years), took part in this study. The New Zealand students (n = 323; female =134, male = 189) came from five secondary schools in the upper half of the North Island of New Zealand, while the

Japanese students (n = 291; female = 131, male =160) came from four junior high schools in the Kanto area in Japan.

In both New Zealand and Japan, data were collected from schools representing the full range of the “ability” spectrum, from what are considered “high level” schools through to those that are considered “lower level” schools. This was done to ensure that the range in students’ abilities in mathematics was represented in the student groups included in the study. In New Zealand, both a top decile school and a bottom decile school (cf. New Zealand Ministry of Education, 2005) were included, as well as three schools in between. The math teachers in the schools that participated were also requested to administer the problems to approximately equal numbers of “high,” “average,” and “low” ability math classes.

Procedure

This study was part of a larger study that also examined students’ views about the use of diagrams in problem solving, and their responses to suggestions to use diagrams when attempting to solve problems they were given. Only procedural details and the students’ performance in the first part of the study, which looked at their spontaneous use of diagrams, will be reported here.

A booklet containing math word problems was given to the participants during their regular math class. In New Zealand, the booklet was written in English, while in Japan it was written in Japanese (various measures were taken to ensure equivalence of the versions). The participants were told that the purpose of the research was to find out how they solved math word problems, and they were requested to show their working throughout.

The booklet contained two problems (Problem 1 and Problem 2), one problem to a page with ample space below each problem for the participants to work out the solution and provide their answer. The participants were told that they had 4 minutes to work on each math problem, and not to go to the next problem/page until instructed to do so. There were two versions of the booklet which differed only in the story context attached to the math word problems (see explanation under Materials below) and, in each class, approximately half of the participants were randomly given one version of the booklet while the other half were given the other version. One version of the booklet contained math word problems involving length, and the other version contained non-length problems. In New Zealand, 173 students received the booklet with length problems, and 150 received the booklets with non-length problems. The corresponding numbers that received each kind of booklet in Japan were 149 and 142 students respectively.

Materials

Different kinds of math problems were used to ensure they represented the variety that students have to deal with in real life. Apart from the length and non-length distinctions noted above, in each version of the booklet, two categories of problems were used: one-object problems and two-object problems (similar to ones used in Uesaka, 2003). Problems 1 and 2 in the two versions of the booklet are shown in Table 1.

Table 1. The Kinds and Categories of Math Word Problems Used

Kind	Category: One-Object	Category: Two-Object
Length	You light a candle and it starts to burn. It burns at a constant rate, which means that it burns at the same speed throughout. After 5 minutes, the candle is 10 centimetres in length. After 7 minutes, it is 6 centimetres. How long does it take for the candle to burn out?	Tom's house and Hannah's house are connected by one road which is 600 metres long. Tom and Hannah talked on the telephone and decided to play with each other. They leave their own house at the same time and start to walk towards the other's house. They meet on the road 5 minutes later. The place where they meet is 100 metres closer to Hannah's house than the half-way point. How fast did Tom walk (per minute)? How fast did Hannah walk (per minute)?
Non-Length	A mouse starts to eat a piece of cheese. The mouse eats at a constant rate, which means it eats at the same speed throughout. After 5 minutes, there are 10 grammes left of the cheese. After 7 minutes, there are 6 grammes left. How long does it take the mouse to finish eating the piece of cheese?	There is a pond that can contain up to 600 litres of water. Now the pond is empty so using two taps, A and B, you start to fill the pond with water. Water comes out of both taps at a constant rate. 5 minutes later, the pond is full. From tap A came 100 litres more water than half of all the water in the pond. How fast did water come out of tap A (per minute)? How fast did water come out of tap B (per minute)?

Scoring

The participants' work in solving the math word problems was scored by the second author: she marked the answers they provided as either correct (P+) or incorrect (P–). She also assessed the students' use of diagrams for each of the problems, and determined in each case whether at least one diagram was used (D+) or no diagram was used at all (D–). For this assessment, a diagram was defined as any representation of the problem other than words (on their own), sentences, or numerical formulas. Tables were counted as diagrams and, for the purposes of this study, a table was defined as a depiction of at least a pair of values arrayed to represent two related variables. Drawings or illustrations that were considered to be unrelated to the problem were categorised as D–. A colleague of the second author's took the role of independent assessor of diagram use in the participants' work. The inter-rater agreement was found to be 96.3%, which the present authors deemed as satisfactory.

The qualities of diagrams constructed by students were also analysed. Two kinds of criteria were used to categorize the diagrams: their structure and the information contained in them (see Table 2). If a diagram represented the situation correctly, then it was placed in the higher category (category A) for the first criterion. Those that did not represent the situation correctly were placed in the lower category (category B). The diagrams were then evaluated as to how much relevant information they contained for the second criterion. For example, if a diagram contained additional inferences from the word problem then it was placed in the highest category (category A); if a diagram contained all of the numbers specified in the problem but no additional inferences were evident then it was placed in the second category (category B); and, going to the low end of the scale, if a diagram contained no relevant numbers at all then it was placed in the lowest category (category E).

The diagrams were divided into high and low quality using a combination of these two criteria. In other words, diagrams that represented the situation correctly and contained more relevant information were categorized as “high quality”, while the rest were categorized as “low quality”. To be more exact, for the diagrams produced for the one-object problems, a diagram was classified as high quality when it had been placed in category A for both structure and content. For the two-object problems, a diagram was classified as high quality when it had been placed in category A for structure and in either category A or category B for content. The decision about where to place the cut-off point for content was made partly in view of the number of diagrams falling into each of the high and low quality categories (i.e., a considerably greater proportion of diagrams produced for the one-object problems contained appropriate inferences compared to those produced for the two-object problems – hence only those that were categorized as A for content were placed in the high quality category where the one-object problems were concerned).

Table 2. The Criteria Used for Evaluating the Quality of Diagrams Students Produced for the One- and Two-Object Problems

Points of Evaluation	Categories	Criteria for Placement in the Categories
Structure of Diagram	A	Represents the situation correctly
	B	Does not represent the situation correctly
Information Contained in Diagram	A	Contains additional inferences drawn from the problem
	B	Contains all of the numbers specified in the problem
	C	Contains some of the numbers specified in the problem
	D	Contains some numbers, but all of them are incorrect or unrelated to the problem
	E	Contains no numbers

Results

The percentages of correct answers provided by the NZ and Japanese students to the math problems given are shown in Table 3. In all four kinds and categories of problems given, the percentages of correct responses provided by the NZ students were significantly higher than the corresponding percentages from the Japanese students.

Table 3. Percentages of Correct Answers from the New Zealand and Japanese Students

Kind of Problem	Category of Problem	Percentages of Correct Answers		$\chi^2_{(1)}$ value
		New Zealand Students	Japanese Students	
Length	One-Object	78.03	49.66	27.29***
	Two-Object	64.74	49.66	7.46**
Non-Length	One-Object	70.00	50.70	11.38***
	Two-Object	65.33	40.85	17.58***

* $p < .05$. ** $p < .01$. *** $p < .001$.

The percentages of NZ and Japanese students who spontaneously used diagrams in attempting to solve each of the four kinds and categories of problems given are shown in Table 4. A significantly higher percentage of the NZ students spontaneously used diagrams in attempting to solve one-object problems of the length and non-length kinds. However, this was not the case as far as the more complex two-object problems were concerned: no significant difference was found between the student groups where the two-object, length problem was concerned, and the Japanese students spontaneously used more diagrams in attempting to solve the two-object, non-length problem.

Table 4. Percentages of New Zealand and Japanese Students Who Spontaneously Used Diagrams in Attempting to Solve the Problems Given

Kind of Problem	Category of Problem	Percentages of Students		$\chi^2_{(1)}$ value
		New Zealand Students	Japanese Students	
Length	One-Object	60.69	37.58	17.10***
	Two-Object	58.96	57.05	0.12
Non-Length	One-Object	48.00	31.43	8.28**
	Two-Object	16.00	26.06	4.46*

* $p < .05$. ** $p < .01$. *** $p < .001$.

These latter findings were somewhat unexpected and prompted the authors to more carefully examine the data that had been collected. This examination revealed that, despite there being no difference between the percentages of NZ and Japanese students who produced diagrams for the two-object, length problem, the percentage of NZ students (38.67%) who used a diagram *and* got the answer correct for this problem was significantly higher compared to the corresponding Japanese students percentage (21.13%), $\chi^2_{(1)} = 10.66$, $p < .01$. Where the two-object, non-length problem was concerned, more careful examination of the data revealed that the percentage of NZ students (53.33%) who *did not* use a diagram but produced the correct answer was significantly higher than the corresponding percentage of Japanese students (33.10%), $\chi^2_{(1)} = 12.15$, $p < .001$. There was also a significantly higher percentage of Japanese students (18.31%) who used a diagram but produced the *wrong* answer for this problem (two-object, non-length) compared to the NZ students (4.00%), $\chi^2_{(1)} = 15.31$, $p < .001$.

Table 5 shows the results of the comparison made of the proportion of high quality diagrams produced by the students from the two countries. As shown, except where the one-object problems with length story context were concerned, the proportion of high quality diagrams produced by students from the two countries can be considered equivalent – as no other significant differences were found. In one-object problems with length story context, more high quality diagrams were produced by the New Zealand students compared to their Japanese counterparts. This result suggests that New Zealand students can draw higher quality diagrams for the problems which can be considered as easiest to solve. However the quality of diagrams produced by students from the two countries did not differ for the more difficult problems.

Table 5. Percentages of High Quality Diagrams Produced by the New Zealand and Japanese Students

Kind of Problem	Category of Problem	Percentages of High Quality Diagrams		$\chi^2_{(1)}$ value
		New Zealand Students	Japanese Students	
Length	One-Object	80.95	60.71	7.76**
	Two-Object	75.49	69.41	0.86
Non-length	One-Object	80.56	65.91	3.12
	Two-Object	41.67	29.73	0.92

* $p < .05$. ** $p < .01$.

To check the validity of the classifications made, the relationship between category placement and correct solutions to the problems was analysed. Chi-test analyses revealed that the diagrams categorized as "high quality" significantly produced more correct answers for the problems given compared to those categorized as "low quality" in both one-object problems (length version: $\chi^2_{(1)} = 41.80$, $p < .01$, non-length version: $\chi^2_{(1)} = 6.28$, $p < .01$) and two-object problems (length version: $\chi^2_{(1)} = 25.43$, $p < .01$, non-length version: $\chi^2_{(1)} = 14.33$, $p < .01$). These analysis results confirm the validity of the classification method employed in the present study.

Discussion

The findings of this study indicate that the NZ students, compared to their Japanese counterparts, were better at solving math word problems: significantly more of the NZ students provided the correct answers to the four problems given. However, the NZ students only spontaneously used more diagrams in attempting to solve the two (length and non-length) one-object problems, and the percentage of high quality diagrams they produced was only significantly higher where the easiest one-object, length problem was concerned. This raises some important questions about what the two groups of students could and could not do well as far as math word problem solving and diagram construction were concerned.

It appears that the NZ students were better at constructing diagrams for the easier math word problems: they not only evidenced greater use of diagrams in attempting to solve the one-object problems (both length and non-length kinds), they also evidenced a higher percentage of high quality diagrams constructed for the one-object, length problem. Their skills in using diagrams to solve the more complex two-object problems were more limited – and perhaps this is to be expected (since, after all, these problems were more complicated and difficult to solve). However, this points to a clear need here for the NZ students to develop skills in constructing and using diagrams to help solve more complex math word problems. This same need for skills development applies to the Japanese students where both simpler and more complex math word problems are concerned. To this end, perhaps the use of strategies such as active comparison and review of lessons learnt from problem solving – which Uesaka and Manalo (2006) reported as being effective in promoting competence in the construction of appropriate diagrams to use – could be trialled and evaluated.

When dealing with the more complex two-object, non-length problems, a higher proportion of the Japanese students used diagrams but produced the wrong answers (suggesting that the diagrams they constructed were inappropriate ones). Thus, although there are clear indicators that some of the Japanese students were well able to construct appropriate high quality diagrams to use in problem solving, skills development in appropriate choice of diagrams to use (an area that both Cox, 1996, and Uesaka, 2003, have identified as being problematic for many students) would likely prove beneficial for many.

One point of interest is that more than half of the NZ students were able to correctly solve the two-object, non-length problem correctly without the use of diagrams, as noted earlier. This suggests a number of things. First, it confirms that construction of diagrams for solving more complex problems is more difficult and requires more attention from teachers if students are to develop the knowledge and skills required. Second, it suggests that at least some of the NZ students were able to effectively mentally visualise the situation depicted in the more complex math word problem, which helped them solve it. Third, it also suggests that at least some of the NZ students were able to employ alternative means of solving the more complex math word problem (e.g., through the use of algebraic equations) when they were unable to construct the appropriate diagrammatic representation.

The final point above raises the possible question of whether diagrams are even necessary in solving more complex math word problems. The answer to this question becomes somewhat obvious when one examines the students' problem solving performance: there is a clear decline in production of correct answers for the more complex two-object, non-length problem (which coincides with a decline in the students' use of diagrams). It is of course only possible to speculate that had more students been able to construct the appropriate diagram to solve the problem in question, more of them would have been able to correctly solve it – but such a speculation would be in line with previous findings noted earlier about the benefits and/or advantages of using diagrams when problem solving (e.g., Cheng, 2004; Hembree, 1992; Mayer, 2003; Novick & Hurley, 2001).

With the performance of the NZ students (compared to the Japanese students) being better on various aspects of problem solving and diagram construction, the findings of this study suggests that the NZ math curriculum's greater emphasis on the problem solving and communications applications of diagrams does promote the development of the desired skills in students. Including the same or similar components in the Japanese math curriculum could therefore be beneficial to developing their students' problem solving capabilities. However, as noted earlier, there are some limitations – for example, with the NZ students' advantage in diagram construction and use being limited to the easier problems. This also is only an initial investigation (on *one* age group of students from only two countries), and further studies are clearly required to more fully understand the kinds of instruction, learning conditions, and motivations that impact on the development of students' problem solving skills.

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